Problem 8

In each of Problems 7 through 12, determine the longest interval in which the given initial value problem is certain to have a unique twice-differentiable solution. Do not attempt to find the solution.

$$(t-1)y'' - 3ty' + 4y = \sin t, \qquad y(-2) = 2, \quad y'(-2) = 1$$

Solution

Divide both sides of the ODE by $t-1$ so that the coefficient of y'' is 1.

$$y'' - \frac{3t}{t-1}y' + \frac{4}{t-1}y = \frac{\sin t}{t-1}$$

There is a point of discontinuity at $t = 1$, which means the interval in which the general solution is unique and twice-differentiable is either $-\infty < t < 1$ or $1 < t < \infty$. Because y and y' are prescribed at $t = -2$, the general solution is unique and twice-differentiable on $-\infty < t < 1$.