Problem 9

In each of Problems 7 through 12, determine the longest interval in which the given initial value problem is certain to have a unique twice-differentiable solution. Do not attempt to find the solution.

$$t(t-4)y'' + 3ty' + 4y = 2, \qquad y(3) = 0, \quad y'(3) = -1$$

Solution

Divide both sides of the ODE by $t(t-4)$ so that the coefficient of y'' is 1.

$$y'' + \frac{3}{t-4}y' + \frac{4}{t(t-4)}y = \frac{2}{t(t-4)}$$

There are points of discontinuity at $t = 0$ and $t = 4$, which means the interval in which the general solution is unique and twice-differentiable is either $-\infty < t < 0$ or $0 < t < 4$ or $4 < t < \infty$. Because y and y' are prescribed at $t = 3$, the general solution is unique and twice-differentiable on $0 < t < 4$.