Problem 11

In each of Problems 7 through 12, determine the longest interval in which the given initial value problem is certain to have a unique twice-differentiable solution. Do not attempt to find the solution.

\[(x - 3)y'' + xy' + (\ln |x|)y = 0, \quad y(1) = 0, \quad y'(1) = 1\]

Solution

Divide both sides of the ODE by \(x - 3\) so that the coefficient of \(y''\) is 1.

\[y'' + \frac{x}{x-3}y' + \frac{\ln |x|}{x-3}y = 0\]

There are points of discontinuity at \(x = 0\) and \(x = 3\), which means the interval in which the general solution is unique and twice-differentiable is either \(-\infty < x < 0\) or \(0 < x < 3\) or \(3 < x < \infty\). Because \(y\) and \(y'\) are prescribed at \(x = 1\), the general solution is unique and twice-differentiable on \(0 < x < 3\).