Problem 12

In each of Problems 7 through 12, determine the longest interval in which the given initial value problem is certain to have a unique twice-differentiable solution. Do not attempt to find the solution.

$$(x-2)y'' + y' + (x-2)(\tan x)y = 0, \qquad y(3) = 1, \quad y'(3) = 2$$

Solution

Divide both sides of the ODE by $x - 2$ so that the coefficient of y'' is 1.

$$y'' + \frac{1}{x-2}y' + \frac{\sin x}{\cos x}y = 0$$

There are points of discontinuity at $x = 2$ and $x = (2n-1)\pi/2$ for $n = 0, \pm 1, \pm 2$. The general solution is unique and twice-differentiable in some interval between two adjacent points. Because y and y' are prescribed at $x = 3$, the general solution is unique and twice-differentiable on $2 < x < 3\pi/2$.