Problem 13

Verify that $y_1(t) = t^2$ and $y_2(t) = t^{-1}$ are two solutions of the differential equation $t^2y'' - 2y = 0$ for $t > 0$. Then show that $y = c_1t^2 + c_2t^{-1}$ is also a solution of this equation for any c_1 and c_2.

Solution

Check that $y_1(t)$ is a solution.

$$t^2y_1'' - 2y_1 \stackrel{?}{=} 0$$

$$t^2\frac{d^2}{dt^2}(t^2) - 2(t^2) \stackrel{?}{=} 0$$

$$t^2(2) - 2(t^2) \stackrel{?}{=} 0$$

$$2t^2 - 2t^2 \stackrel{?}{=} 0$$

$$0 = 0$$

Now check that $y_2(t)$ is a solution.

$$t^2y_2'' - 2y_2 \stackrel{?}{=} 0$$

$$t^2\frac{d^2}{dt^2}(t^{-1}) - 2(t^{-1}) \stackrel{?}{=} 0$$

$$t^2[-(-2)t^{-3}] - 2t^{-1} \stackrel{?}{=} 0$$

$$2t^{-1} - 2t^{-1} \stackrel{?}{=} 0$$

$$0 = 0$$

Now check that $y(t)$ is a solution.

$$t^2y'' - 2y \stackrel{?}{=} 0$$

$$t^2\frac{d^2}{dt^2}(c_1t^2 + c_2t^{-1}) - 2(c_1t^2 + c_2t^{-1}) \stackrel{?}{=} 0$$

$$t^2[2c_1 - (-2)t^{-3}] - 2c_1t^2 - 2c_2t^{-1} \stackrel{?}{=} 0$$

$$2c_1t^2 + 2t^{-1} - 2c_1t^2 - 2c_2t^{-1} \stackrel{?}{=} 0$$

$$0 = 0$$