

## Problem 19

If  $W(f, g)$  is the Wronskian of  $f$  and  $g$ , and if  $u = 2f - g$ ,  $v = f + 2g$ , find the Wronskian  $W(u, v)$  of  $u$  and  $v$  in terms of  $W(f, g)$ .

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### Solution

Suppose  $W(f, g)$  is the Wronskian of  $f$  and  $g$ . Then

$$W(f, g) = \begin{vmatrix} f & g \\ f' & g' \end{vmatrix} = fg' - gf'.$$

Now consider  $W(u, v)$ , the Wronskian of  $u$  and  $v$ .

$$\begin{aligned} W(u, v) &= \begin{vmatrix} u & v \\ u' & v' \end{vmatrix} \\ &= \begin{vmatrix} 2f - g & f + 2g \\ 2f' - g' & f' + 2g' \end{vmatrix} \\ &= (2f - g)(f' + 2g') - (f + 2g)(2f' - g') \\ &= \cancel{2ff'} + 4fg' - f'g - \cancel{2gg'} - \cancel{2ff'} + fg' - 4f'g + \cancel{2gg'} \\ &= 5fg' - 5f'g \\ &= 5(fg' - f'g) \end{aligned}$$

Therefore,

$$W(u, v) = 5W(f, g).$$