Problem 14

In each of Problems 7 through 16, find the general solution of the given differential equation.

\[ 9y'' + 9y' - 4y = 0 \]

Solution

Since this is a linear homogeneous constant-coefficient ODE, the solution is of the form \( y = e^{rt} \).

\[ \begin{align*}
    y &= e^{rt} \\
    y' &= re^{rt} \\
    y'' &= r^2 e^{rt}
\end{align*} \]

Substitute these expressions into the ODE.

\[ 9(r^2 e^{rt}) + 9(re^{rt}) - 4(e^{rt}) = 0 \]

Divide both sides by \( e^{rt} \).

\[ 9r^2 + 9r - 4 = 0 \]

\[ (3r + 4)(3r - 1) = 0 \]

\[ r = \left\{ \begin{array}{l}
    -\frac{4}{3} \\
    \frac{1}{3}
\end{array} \right\} \]

Two solutions to the ODE are \( y = e^{-4t/3} \) and \( y = e^{t/3} \), so the general solution is

\[ y(t) = C_1 e^{-4t/3} + C_2 e^{t/3}, \]

a linear combination of the two.