Problem 15

In each of Problems 7 through 16, find the general solution of the given differential equation.

$$y'' + y' + 1.25y = 0$$

Solution

Since this is a linear homogeneous constant-coefficient ODE, the solution is of the form $y = e^{rt}$.

$$y = e^{rt} \quad \rightarrow \quad y' = re^{rt} \quad \rightarrow \quad y'' = r^2e^{rt}$$

Substitute these expressions into the ODE.

$$r^2e^{rt} + re^{rt} + 1.25(e^{rt}) = 0$$

Divide both sides by e^{rt}.

$$r^2 + r + 1.25 = 0$$

$$r = \frac{-1 \pm \sqrt{1 - 4(1.25)(1)}}{2} = \frac{-1 \pm \sqrt{-4}}{2} = \frac{-1 \pm 2i}{2} = -\frac{1}{2} \pm i$$

$$r = \left\{-\frac{1}{2} - i, -\frac{1}{2} + i\right\}$$

Two solutions to the ODE are $y = e^{(-1/2-i)t}$ and $y = e^{(-1/2+i)t}$, so the general solution is a linear combination of the two.

$$\begin{aligned}
y(t) &= C_1e^{(-1/2-i)t} + C_2e^{(-1/2+i)t} \\
&= C_1e^{-t/2-it} + C_2e^{-t/2+it} \\
&= C_1e^{-t/2}e^{-it} + C_2e^{-t/2}e^{it} \\
&= C_1e^{-t/2}[\cos(-t) + i\sin(-t)] + C_2e^{-t/2}(\cos t + i\sin t) \\
&= C_1e^{-t/2}(\cos t - i\sin t) + C_2e^{-t/2}(\cos t + i\sin t) \\
&= C_1e^{-t/2}\cos t - iC_1e^{-t/2}\sin t + C_2e^{-t/2}\cos t + iC_2e^{-t/2}\sin t \\
&= (C_1 + C_2)e^{-t/2}\cos t + (-iC_1 + iC_2)e^{-t/2}\sin t
\end{aligned}$$

Therefore, using C_3 for $C_1 + C_2$ and C_4 for $-iC_1 + iC_2$, the real general solution is

$$y(t) = C_3e^{-t/2}\cos t + C_4e^{-t/2}\sin t.$$