Problem 3

In each of Problems 1 through 10, find the general solution of the given differential equation.

$$4y'' - 4y' - 3y = 0$$

Solution

Since this is a linear homogeneous constant-coefficient ODE, the solution is of the form $y = e^{rt}$.

$$y = e^{rt} \quad \to \quad y' = re^{rt} \quad \to \quad y'' = r^2 e^{rt}$$

Substitute these expressions into the ODE.

$$4(r^2 e^{rt}) - 4(re^{rt}) - 3(e^{rt}) = 0$$

Divide both sides by e^{rt}.

$$4r^2 - 4r - 3 = 0$$

$$(2r - 3)(2r + 1) = 0$$

$$r = \left\{-\frac{1}{2}, \frac{3}{2}\right\}$$

Two solutions to the ODE are $y = e^{(-1/2)t}$ and $y = e^{(3/2)t}$, so the general solution is a linear combination of the two.

$$y(t) = C_1 e^{(-1/2)t} + C_2 e^{(3/2)t}$$

Therefore,

$$y(t) = C_1 e^{-t/2} + C_2 e^{3t/2}.$$