Problem 9

In each of Problems 1 through 10, find the general solution of the given differential equation.

\[ 25y'' - 20y' + 4y = 0 \]

Solution

Since this is a linear homogeneous constant-coefficient ODE, the solution is of the form \( y = e^{rt} \).

\[
y = e^{rt} \quad \rightarrow \quad y' = re^{rt} \quad \rightarrow \quad y'' = r^2 e^{rt}
\]

Substitute these expressions into the ODE.

\[
25(r^2 e^{rt}) - 20(re^{rt}) + 4(e^{rt}) = 0
\]

Divide both sides by \( e^{rt} \).

\[
25r^2 - 20r + 4 = 0
\]

\[
(5r - 2)^2 = 0
\]

\[
r = \left\{ \frac{2}{5} \right\}
\]

One solution to the ODE is \( y = e^{2t/5} \). Because the ODE is homogeneous, any constant multiple of this is also a solution, that is, \( y = ce^{2t/5} \). According to the method of reduction of order, the general solution is found by allowing \( c \) to vary as a function of \( t \).

\[
y(t) = c(t)e^{2t/5}
\]

Substitute this expression for \( y \) into the original ODE to determine \( c(t) \).

\[
25y'' - 20y' + 4y = 0 \quad \rightarrow \quad 25[c(t)e^{2t/5}]'' - 20[c(t)e^{2t/5}]' + 4[c(t)e^{2t/5}] = 0
\]

Evaluate the derivatives using the product rule.

\[
25 \left[ c'(t)e^{2t/5} + \frac{2}{5}c(t)e^{2t/5} \right]' - 20 \left[ c'(t)e^{2t/5} + \frac{2}{5}c(t)e^{2t/5} \right] + 4[c(t)e^{2t/5}] = 0
\]

\[
25c''(t)e^{2t/5} + 10c'(t)e^{2t/5} + \frac{4}{5}c(t)e^{2t/5} - 20c'(t)e^{2t/5} - 8c(t)e^{2t/5} + 4c(t)e^{2t/5} = 0
\]

\[
25c''(t)e^{2t/5} = 0
\]

Divide both sides by \( 25e^{2t/5} \).

\[
c''(t) = 0
\]

Integrate both sides with respect to \( t \).

\[
c'(t) = C_1
\]

Integrate both sides with respect to \( t \) once more.

\[
c(t) = C_1t + C_2
\]

Therefore, the general solution is

\[
y(t) = C_1te^{2t/5} + C_2e^{2t/5}.
\]

www.stemjock.com