# Problem 1

In each of Problems 1 through 4, determine $\omega_0$, $R$, and $\delta$ so as to write the given expression in the form $u = R\cos(\omega_0 t - \delta)$.

$$u = 3\cos 2t + 4\sin 2t$$

---

**Solution**

We wish to write the two sinusoidal terms as one.

$$\begin{aligned} 3\cos 2t + 4\sin 2t &= R\cos(\omega_0 t - \delta) \\ &= R[\cos\omega_0 t\cos\delta + \sin\omega_0 t\sin\delta] \\ &= (R\cos\delta)\cos\omega_0 t + (R\sin\delta)\sin\omega_0 t \end{aligned}$$

Matching the coefficients, we obtain the following system of equations for $\omega_0$, $R$, and $\delta$.

$$R\cos\delta = 3 \tag{1}$$
$$\omega_0 = 2 \tag{2}$$
$$R\sin\delta = 4 \tag{3}$$

Square both sides of the first and third equations

$$R^2\cos^2\delta = 9$$
$$R^2\sin^2\delta = 16$$

and add their respective sides.

$$R^2\cos^2\delta + R^2\sin^2\delta = 9 + 16$$
$$R^2(\cos^2\delta + \sin^2\delta) = 25$$
$$R^2 = 25$$
$$R = 5$$

Divide the respective sides of equations (1) and (3).

$$\frac{R\sin\delta}{R\cos\delta} = \frac{4}{3} \quad \to \quad \tan\delta = \frac{4}{3} \quad \to \quad \delta = \tan^{-1}\frac{4}{3}$$

Therefore,

$$3\cos 2t + 4\sin 2t = 5\cos\left(2t - \tan^{-1}\frac{4}{3}\right).$$