

## Problem 2

In each of Problems 1 through 4, determine  $\omega_0$ ,  $R$ , and  $\delta$  so as to write the given expression in the form  $u = R \cos(\omega_0 t - \delta)$ .

$$u = -\cos t + \sqrt{3} \sin t$$

### Solution

We wish to write the two sinusoidal terms as one.

$$\begin{aligned} -\cos t + \sqrt{3} \sin t &= R \cos(\omega_0 t - \delta) \\ &= R[\cos \omega_0 t \cos \delta + \sin \omega_0 t \sin \delta] \\ &= (R \cos \delta) \cos \omega_0 t + (R \sin \delta) \sin \omega_0 t \end{aligned}$$

Matching the coefficients, we obtain the following system of equations for  $\omega_0$ ,  $R$ , and  $\delta$ .

$$R \cos \delta = -1 \tag{1}$$

$$\omega_0 = 1 \tag{2}$$

$$R \sin \delta = \sqrt{3} \tag{3}$$

Square both sides of the first and third equations

$$R^2 \cos^2 \delta = 1$$

$$R^2 \sin^2 \delta = 3$$

and add their respective sides.

$$R^2 \cos^2 \delta + R^2 \sin^2 \delta = 1 + 3$$

$$R^2(\cos^2 \delta + \sin^2 \delta) = 4$$

$$R^2 = 4$$

$$R = 2$$

Divide the respective sides of equations (1) and (3).

$$\frac{R \sin \delta}{R \cos \delta} = \frac{\sqrt{3}}{-1} \rightarrow \tan \delta = -\sqrt{3} \rightarrow \delta = \tan^{-1}(-\sqrt{3}) = -\tan^{-1} \sqrt{3} = -\frac{\pi}{3}$$

Therefore,

$$-\cos t + \sqrt{3} \sin t = 2 \cos \left( t + \frac{\pi}{3} \right).$$