Problem 2

In each of Problems 1 through 4, determine ω_0, R, and δ so as to write the given expression in the form $u = R\cos(\omega_0 t - \delta)$.

$$u = -\cos t + \sqrt{3}\sin t$$

Solution

We wish to write the two sinusoidal terms as one.

$$\begin{aligned} -\cos t + \sqrt{3}\sin t &= R\cos(\omega_0 t - \delta) \\ &= R[\cos\omega_0 t\cos\delta + \sin\omega_0 t\sin\delta] \\ &= (R\cos\delta)\cos\omega_0 t + (R\sin\delta)\sin\omega_0 t \end{aligned}$$

Matching the coefficients, we obtain the following system of equations for ω_0, R, and δ.

$$R\cos\delta = -1 \tag{1}$$
$$\omega_0 = 1 \tag{2}$$
$$R\sin\delta = \sqrt{3} \tag{3}$$

Square both sides of the first and third equations

$$R^2\cos^2\delta = 1$$
$$R^2\sin^2\delta = 3$$

and add their respective sides.

$$R^2\cos^2\delta + R^2\sin^2\delta = 1 + 3$$
$$R^2(\cos^2\delta + \sin^2\delta) = 4$$
$$R^2 = 4$$
$$R = 2$$

Divide the respective sides of equations (1) and (3).

$$\frac{R\sin\delta}{R\cos\delta} = \frac{\sqrt{3}}{-1} \quad \to \quad \tan\delta = -\sqrt{3} \quad \to \quad \delta = \tan^{-1}(-\sqrt{3}) + \pi = -\tan^{-1}\sqrt{3} + \pi = -\frac{\pi}{3} + \pi = \frac{2\pi}{3}$$

Therefore,

$$-\cos t + \sqrt{3}\sin t = 2\cos\left(t - \frac{2\pi}{3}\right).$$