Problem 4

In each of Problems 1 through 4, determine ω_0, R, and δ so as to write the given expression in the form $u = R\cos(\omega_0 t - \delta)$.

$$u = -2\cos\pi t - 3\sin\pi t$$

Solution

We wish to write the two sinusoidal terms as one.

$$\begin{aligned} -2\cos\pi t - 3\sin\pi t &= R\cos(\omega_0 t - \delta) \\ &= R[\cos\omega_0 t\cos\delta + \sin\omega_0 t\sin\delta] \\ &= (R\cos\delta)\cos\omega_0 t + (R\sin\delta)\sin\omega_0 t \end{aligned}$$

Matching the coefficients, we obtain the following system of equations for ω_0, R, and δ.

$$R\cos\delta = -2 \tag{1}$$

$$\omega_0 = \pi \tag{2}$$

$$R\sin\delta = -3 \tag{3}$$

Square both sides of the first and third equations

$$\begin{aligned} R^2\cos^2\delta &= 4 \\ R^2\sin^2\delta &= 9 \end{aligned}$$

and add their respective sides.

$$\begin{aligned} R^2\cos^2\delta + R^2\sin^2\delta &= 4 + 9 \\ R^2(\cos^2\delta + \sin^2\delta) &= 13 \\ R^2 &= 13 \\ R &= \sqrt{13} \end{aligned}$$

Divide the respective sides of equations (1) and (3).

$$\frac{R\sin\delta}{R\cos\delta} = \frac{-3}{-2} \quad \rightarrow \quad \tan\delta = \frac{3}{2} \quad \rightarrow \quad \delta = \tan^{-1}\frac{3}{2} + \pi$$

Therefore,

$$\begin{aligned} -2\cos\pi t - 3\sin\pi t &= \sqrt{13}\cos\left(\pi t - \tan^{-1}\frac{3}{2} - \pi\right) \\ &= -\sqrt{13}\cos\left(\pi t - \tan^{-1}\frac{3}{2}\right). \end{aligned}$$