Problem 22

Referring to Problem 21, find the logarithmic decrement of the system in Problem 10.

Solution

In Problem 10 the system described by $mx'' + cx' + kx = W$ with the initial conditions, $x(0) = 0.25$ and $x'(0) = 0.25$, was considered, and the result found was (in feet)

$$\begin{aligned} x(t) &= \frac{0.25}{\mu} e^{-ct/2m} \sin \mu t + \frac{W}{k} \\ &= \frac{m}{2\sqrt{4mk - c^2}} e^{-ct/2m} \sin \frac{\sqrt{4mk - c^2}}{2m} t + \frac{W}{k}, \end{aligned}$$

where

$$\begin{aligned} k &= 64 \ \frac{\text{lb}}{\text{ft}} \\ c &= 2 \ \frac{\text{lb} \cdot \text{s}}{\text{ft}} \\ W &= 16 \ \text{lb} \\ m &= \frac{W}{g} = \frac{16 \ \text{lb}}{32.2 \ \frac{\text{ft}}{\text{s}^2}} \approx 0.5. \end{aligned}$$

The logarithmic decrement is

$$\begin{aligned} \Delta &= \frac{\pi c}{m\mu} \\ &= \frac{\pi c}{m \frac{\sqrt{4mk - c^2}}{2m}} \\ &= \frac{2\pi c}{\sqrt{4mk - c^2}} \\ &\approx 1.13. \end{aligned}$$