Problem 2

In each of Problems 1 through 4, determine $\omega_0$, $R$, and $\delta$ so as to write the given expression in the form $u = R \cos(\omega_0 t - \delta)$.

$$u = -\cos t + \sqrt{3} \sin t$$

Solution

We wish to write the two sinusoidal terms as one.

$$-\cos t + \sqrt{3} \sin t = R \cos(\omega_0 t - \delta)$$

$$= R \cos \omega_0 t \cos \delta + \sin \omega_0 t \sin \delta$$

$$= (R \cos \delta) \cos \omega_0 t + (R \sin \delta) \sin \omega_0 t$$

Matching the coefficients, we obtain the following system of equations for $\omega_0$, $R$, and $\delta$.

\[
\begin{align*}
R \cos \delta &= -1 \\
\omega_0 &= 1 \\
R \sin \delta &= \sqrt{3}
\end{align*}
\]

(1) (2) (3)

Square both sides of the first and third equations

$$R^2 \cos^2 \delta = 1$$

$$R^2 \sin^2 \delta = 3$$

and add their respective sides.

$$R^2 \cos^2 \delta + R^2 \sin^2 \delta = 1 + 3$$

$$R^2 (\cos^2 \delta + \sin^2 \delta) = 4$$

$$R^2 = 4$$

$$R = 2$$

Divide the respective sides of equations (1) and (3).

$$\frac{R \sin \delta}{R \cos \delta} = \frac{\sqrt{3}}{-1} \rightarrow \tan \delta = -\sqrt{3} \rightarrow \delta = \tan^{-1}(-\sqrt{3}) = -\tan^{-1}\sqrt{3} = -\frac{\pi}{3}$$

Therefore,

$$-\cos t + \sqrt{3} \sin t = 2 \cos \left(t + \frac{\pi}{3}\right).$$