Problem 28

The position of a certain undamped spring-mass system satisfies the initial value problem

\[ u'' + 2u = 0, \quad u(0) = 0, \quad u'(0) = 2. \]

(a) Find the solution of this initial value problem.

(b) Plot \( u \) versus \( t \) and \( u' \) versus \( t \) on the same axes.

(c) Plot \( u' \) versus \( u \); that is, plot \( u(t) \) and \( u'(t) \) parametrically with \( t \) as the parameter. This plot is known as a phase plot, and the \( uu' \)-plane is called the phase plane. Observe that a closed curve in the phase plane corresponds to a periodic solution \( u(t) \). What is the direction of motion on the phase plot as \( t \) increases?