

Problem 20

For the initial value problem in Problem 18, plot u' versus u for $\omega = 0.7$, $\omega = 0.8$, and $\omega = 0.9$. Such a plot is called a phase plot. Use a t interval that is long enough so that the phase plot appears as a closed curve. Mark your curve with arrows to show the direction in which it is traversed as t increases.

Solution

The initial value problem of Problem 18 was

$$u'' + u = 3 \cos \omega t, \quad u(0) = 0, \quad u'(0) = 0,$$

and its solution was found to be

$$u(t) = \frac{3}{1 - \omega^2} (\cos \omega t - \cos t).$$

Differentiate it with respect to t .

$$u'(t) = \frac{3}{1 - \omega^2} (-\omega \sin \omega t + \sin t)$$

Phase plots, that is, parametric plots of

$$\left(\frac{3}{1 - \omega^2} (\cos \omega t - \cos t), \frac{3}{1 - \omega^2} (-\omega \sin \omega t + \sin t) \right)$$

for various values of ω are shown below.





