Problem 25

Suppose that the system of Problem 24 is modified to include a damping term and that the resulting initial value problem is

$$u'' + \frac{1}{5}u' + u + \frac{1}{5}u^3 = \cos\omega t, \qquad u(0) = 0, \quad u'(0) = 0.$$

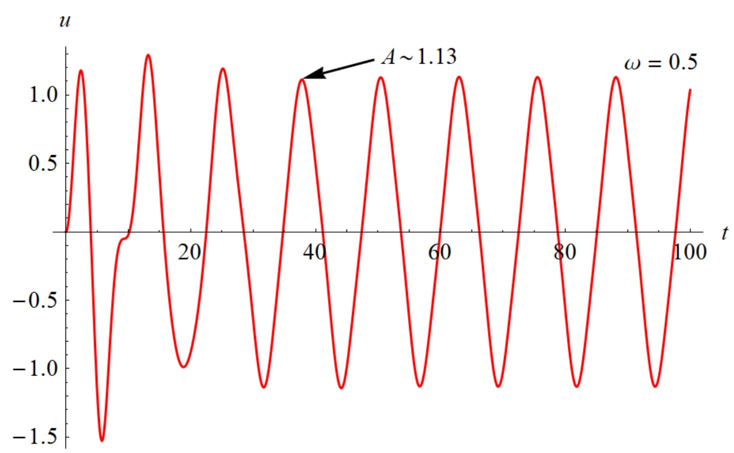
(a) Plot a computer-generated solution of the given problem for several values of ω between 1/2 and 2, and estimate the amplitude R of the steady response in each case.

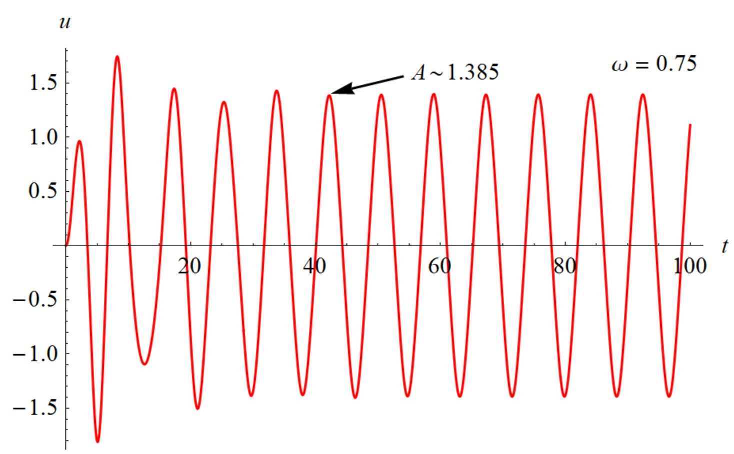
(b) Using the data from part (a), plot the graph of R versus ω. For what frequency ω is the amplitude greatest?

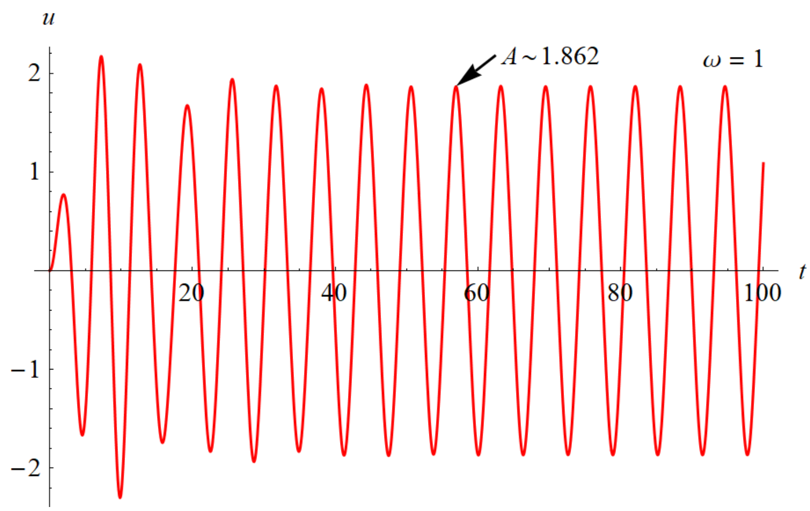
(c) Compare the results of parts (a) and (b) with the corresponding results for the linear spring.

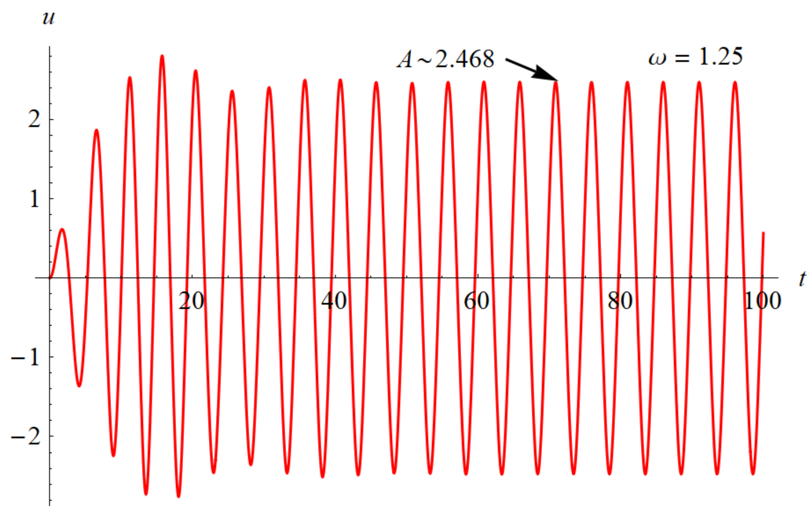
Solution

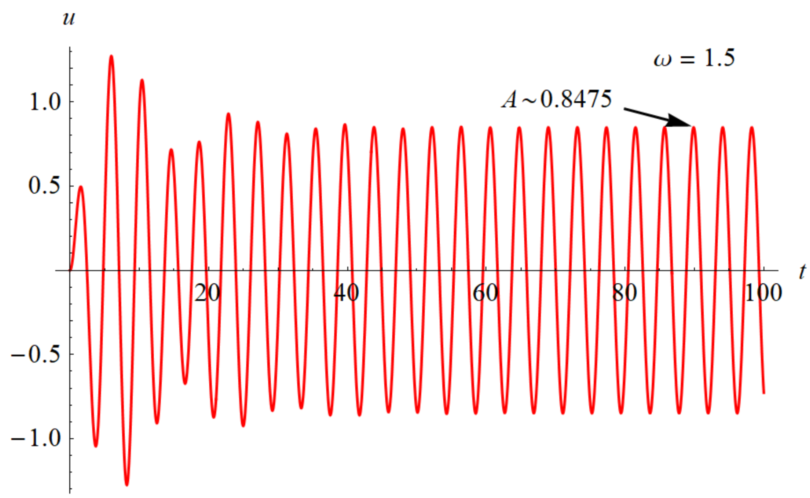
Part (a)

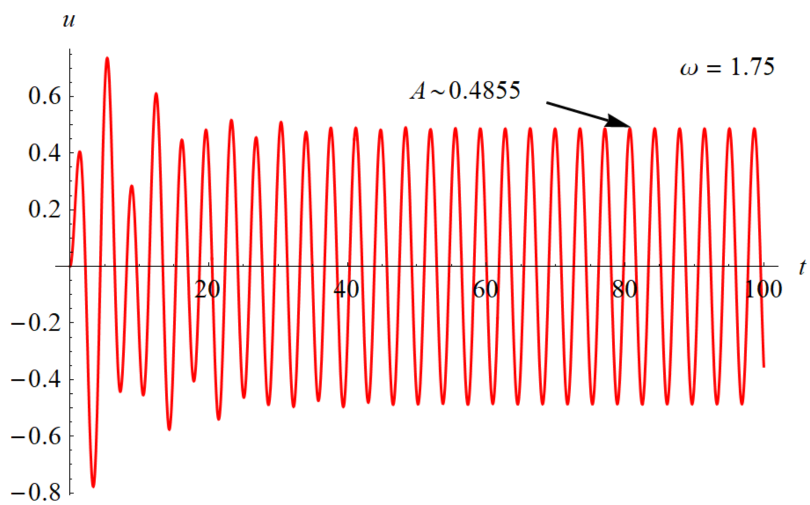


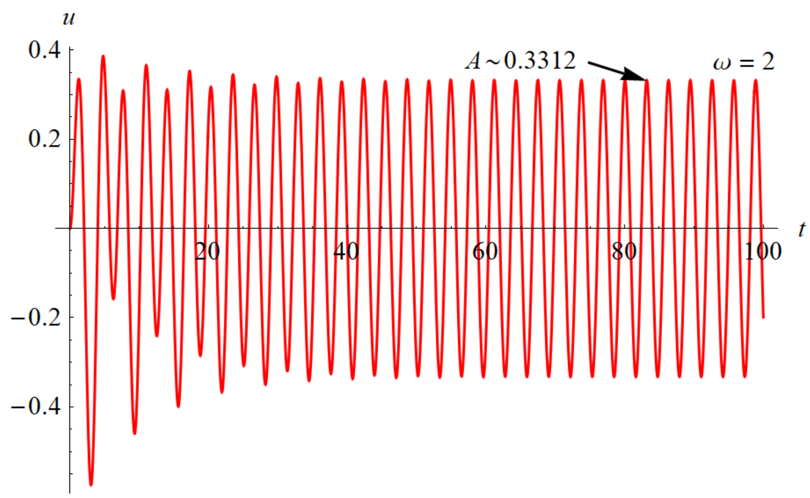



u
A ~ 1.862
ω = 1
2
1
−1
−2
20
40
60
80
100
t

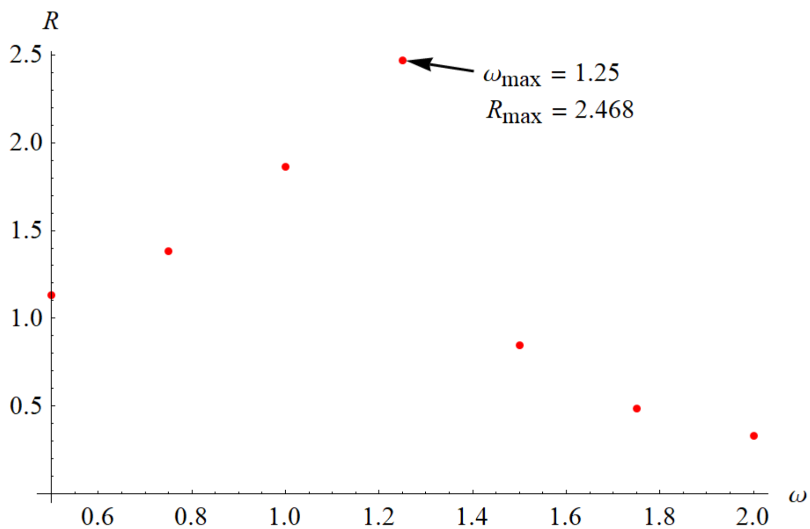

u
A ~ 2.468
ω = 1.25
2
1
−1
−2
20
40
60
80
100
t


u
ω = 1.5
A ~ 0.8475
1.0
0.5
20
40
60
80
100
t
−0.5
−1.0


u
ω = 1.75
A ~ 0.4855
0.6
0.4
0.2
20
40
60
80
100
t
−0.2
−0.4
−0.6
−0.8


u
0.4
0.2
−0.2
−0.4
A ~ 0.3312
ω = 2
t
20
40
60
80
100

Part (b)


R
2.5
2.0
1.5
1.0
0.5
ω_max = 1.25
R_max = 2.468
ω
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0

Part (c)

The amplitude of the steady state for the corresponding linear system described by

$$mu'' + \gamma u' + ku = F_0 \cos \omega t$$

is given by

$$R(\omega) = \frac{F_0}{\sqrt{m^2(\omega_0^2 - \omega^2) + \gamma^2\omega^2}},$$

where $\omega_0^2 = k/m$. For this problem, $m = 1$ and $\gamma = 0.2$ and $k = 1$ and $F_0 = 1$. This function for $R(\omega)$ is plotted below in blue, and it is superimposed on the previous plot of R versus ω.