Problem 14

Find the velocity of the steady state response given by Eq. (10). Then show that the velocity is maximum when \( \omega = \omega_0 \).

Solution

The steady-state response is given by Eq. (10) in the text.

\[
U(t) = R \cos(\omega t - \delta) \quad (10)
\]

The velocity is obtained by differentiating it with respect to \( t \).

\[
U'(t) = -R \omega \sin(\omega t - \delta)
\]

Substitute the formula found for \( R \).

\[
U'(t) = -\frac{F_0}{\sqrt{m^2(\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2}} \omega \sin(\omega t - \delta)
\]

The velocity is maximized when the denominator is minimized, which occurs when \( \omega = \omega_0 \).