Problem 3

In each of Problems 1 through 4, write the given expression as a product of two trigonometric functions of different frequencies.

\[ \cos \pi t + \cos 2\pi t \]

Solution

Recall the sum-to-product formula,

\[ \cos u + \cos v = 2 \cos \left( \frac{u + v}{2} \right) \cos \left( \frac{u - v}{2} \right). \]

Using this, the given expression becomes

\[
\begin{align*}
\cos \pi t + \cos 2\pi t &= 2 \cos \left( \frac{\pi t + 2\pi t}{2} \right) \cos \left( \frac{\pi t - 2\pi t}{2} \right) \\
&= 2 \cos \frac{3\pi t}{2} \cos \frac{\pi t}{2}.
\end{align*}
\]