Problem 7

(a) Find the solution of Problem 5.

(b) Plot the graph of the solution.

(c) If the given external force is replaced by a force \(4 \sin \omega t\) of frequency \(\omega\), find the value of \(\omega\) for which resonance occurs.

Solution

The initial value problem in Problem 5 was

\[ mx'' + kx = W + 2 \cos 3t, \quad x(0) = \frac{7}{24}, \quad x'(0) = 0, \]

where

\[ m = \frac{4 \text{ lb}}{32.2 \frac{\text{ft}}{\text{s}^2}} \]
\[ k = 32 \frac{\text{lb}}{\text{ft}} \]
\[ W = 4 \text{ lb}, \]

and its solution was found to be

\[ x(t) = \left( \frac{7}{24} - \frac{W}{k} - \frac{2}{k - 9m} \right) \cos \sqrt{\frac{k}{m} t} + \frac{W}{k} + \frac{2}{k - 9m} \cos 3t. \]

As \(x(t)\) is in feet, multiply the result by 12 to convert it to inches.

Suppose now that the external force is \(4 \sin \omega t\) rather than \(2 \cos 3t\). Resonance occurs when \(\omega\) matches the natural frequency of the system, that is,

\[ \omega = \sqrt{\frac{k}{m}} \approx 16.05 \frac{\text{rad}}{\text{s}}. \]

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