

## Problem 8

In each of Problems 7 through 10, determine whether the given functions are linearly dependent or linearly independent. If they are linearly dependent, find a linear relation among them.

$$f_1(t) = 2t - 3, \quad f_2(t) = 2t^2 + 1, \quad f_3(t) = 3t^2 + t$$

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### Solution

Form the linear combination of  $f_1$ ,  $f_2$ , and  $f_3$ .

$$C_1f_1 + C_2f_2 + C_3f_3 = C_1(2t - 3) + C_2(2t^2 + 1) + C_3(3t^2 + t)$$

Set it equal to zero.

$$C_1(2t - 3) + C_2(2t^2 + 1) + C_3(3t^2 + t) = 0 \tag{1}$$

$$(-3C_1 + C_2) + (2C_1 + C_3)t + (2C_2 + 3C_3)t^2 = 0 + 0t + 0t^2$$

Match the coefficients.

$$-3C_1 + C_2 = 0$$

$$2C_1 + C_3 = 0$$

$$2C_2 + 3C_3 = 0$$

Solving this system of equations yields  $C_2 = 3C_1$  and  $C_3 = -2C_1$ , where  $C_1$  is arbitrary. Since equation (1) can be satisfied by setting  $C_1 = 1$  and  $C_2 = 3$  and  $C_3 = -2$ , for example,  $f_1$  and  $f_2$  and  $f_3$  are linearly dependent.