

Problem 11

In each of Problems 11 through 16, verify that the given functions are solutions of the differential equation, and determine their Wronskian.

$$y''' + y' = 0; \quad 1, \quad \cos t, \quad \sin t$$

Solution

Check that the first solution satisfies the ODE.

$$(1)''' + (1)' \stackrel{?}{=} 0$$

$$(0) + (0) \stackrel{?}{=} 0$$

$$0 = 0$$

Now check that the second solution satisfies the ODE.

$$(\cos t)''' + (\cos t)' \stackrel{?}{=} 0$$

$$(\sin t) + (-\sin t) \stackrel{?}{=} 0$$

$$0 = 0$$

Now check that the third solution satisfies the ODE.

$$(\sin t)''' + (\sin t)' \stackrel{?}{=} 0$$

$$(-\cos t) + (\cos t) \stackrel{?}{=} 0$$

$$0 = 0$$

The Wronskian of the three functions is

$$W(1, \cos t, \sin t) = \begin{vmatrix} 1 & \cos t & \sin t \\ (1)' & (\cos t)' & (\sin t)' \\ (1)'' & (\cos t)'' & (\sin t)'' \end{vmatrix} = \begin{vmatrix} 1 & \cos t & \sin t \\ 0 & -\sin t & \cos t \\ 0 & -\cos t & -\sin t \end{vmatrix} = 1(\sin^2 t + \cos^2 t) = 1.$$