Problem 19

Let the linear differential operator \( L \) be defined by

\[
L[y] = a_0 y^{(n)} + a_1 y^{(n-1)} + \cdots + a_n y,
\]

where \( a_0, a_1, \ldots, a_n \) are real constants.

(a) Find \( L[t^n] \).

(b) Find \( L[e^{rt}] \).

(c) Determine four solutions of the equation \( y^{(4)} - 5y'' + 4y = 0 \). Do you think the four solutions form a fundamental set of solutions? Why?