Problem 3

In each of Problems 1 through 6, determine intervals in which solutions are sure to exist.

\[ t(t - 1)y^{(4)} + e^t y'' + 4t^2 y = 0 \]

Solution

Divide both sides by \( t(t - 1) \) so that the coefficient of the highest derivative is 1.

\[ y^{(4)} + \frac{e^t}{t(t - 1)} y'' + \frac{4t}{t - 1} y = 0 \]

Two points of discontinuity are \( t = 0 \) and \( t = 1 \), so depending when the initial conditions are given, the solution to this ODE will be valid either for \(-\infty < t < 0\) or \(0 < t < 1\) or \(1 < t < \infty\).