

Problem 8

In each of Problems 7 through 10, follow the procedure illustrated in Example 4 to determine the indicated roots of the given complex number.

$$(1 - i)^{1/2}$$

Solution

Write $1 - i$ in the form of $Re^{i\theta}$.

$$\begin{aligned}(1 - i)^{1/2} &= [\sqrt{2}e^{i(-\pi/4+2n\pi)}]^{1/2}, \quad n = 0, \pm 1, \pm 2, \dots \\ &= \sqrt[4]{2}e^{i(-\pi/8+n\pi)}\end{aligned}$$

The two distinct roots are obtained by setting $n = 0$ and $n = 1$. Other values of n lead to redundant roots.

$$n = 0: \quad (1 - i)^{1/2} = \sqrt[4]{2}e^{-i\pi/8} = \sqrt[4]{2} \left(\cos \frac{\pi}{8} - i \sin \frac{\pi}{8} \right) \approx 1.099 - 0.455i$$

$$n = 1: \quad (1 - i)^{1/2} = \sqrt[4]{2}e^{7i\pi/8} = \sqrt[4]{2} \left(\cos \frac{7\pi}{8} + i \sin \frac{7\pi}{8} \right) \approx -1.099 + 0.455i$$