Problem 16

In each of Problems 11 through 28, find the general solution of the given differential equation.

$$y^{(4)} - 5y'' + 4y = 0$$

Solution

This is a homogeneous ODE with constant coefficients, so the solution is of the form $y = e^{rt}$.

$$y = e^{rt} \quad \rightarrow \quad y' = re^{rt} \quad \rightarrow \quad y'' = r^2e^{rt} \quad \rightarrow \quad y''' = r^3e^{rt} \quad \rightarrow \quad y^{(4)} = r^4e^{rt}$$

Substitute these expressions into the ODE.

$$r^4e^{rt} - 5(r^2e^{rt}) + 4(e^{rt}) = 0$$

Divide both sides by e^{rt}.

$$r^4 - 5r^2 + 4 = 0$$

$$(r^2 - 4)(r^2 - 1) = 0$$

$$(r + 2)(r - 2)(r + 1)(r - 1) = 0$$

$$r = \{-2, -1, 1, 2\}$$

Four solutions to the ODE are then $y = e^{-2t}$ and $y = e^{-t}$ and $y = e^t$ and $y = e^{2t}$. By the principle of superposition, the general solution is a linear combination of these four. Therefore,

$$y(t) = C_1e^{-2t} + C_2e^{-t} + C_3e^t + C_4e^{2t}.$$