Problem 10

In each of Problems 7 through 10, follow the procedure illustrated in Example 4 to determine the indicated roots of the given complex number.

\[ [2(\cos \pi/3 + i \sin \pi/3)]^{1/2} \]

Solution

Write sine and cosine in terms of an exponential function by using Euler’s formula.

\[ [2(\cos \pi/3 + i \sin \pi/3)]^{1/2} = (2e^{i\pi/3})^{1/2} = [2e^{i(\pi/3 + 2n\pi)}]^{1/2}, \quad n = 0, \pm 1, \pm 2, \ldots \]

\[ = \sqrt{2}e^{i(\pi/6 + n\pi)} \]

The two distinct roots are obtained by setting \( n = 0 \) and \( n = 1 \). Other values of \( n \) lead to redundant roots.

\[ n = 0 : \quad [2(\cos \pi/3 + i \sin \pi/3)]^{1/2} = \sqrt{2}e^{i\pi/6} = \sqrt{2}\left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} + \frac{i}{2} \]

\[ n = 1 : \quad [2(\cos \pi/3 + i \sin \pi/3)]^{1/2} = \sqrt{2}e^{7i\pi/6} = \sqrt{2}\left(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6}\right) = -\frac{\sqrt{3}}{2} - \frac{i}{2} \]