Problem 5

In each of Problems 1 through 6, express the given complex number in the form
\[ R(e^{i\theta}) = \sqrt{3} - i \]

Solution

Use Euler’s formula to write \( e^{i\theta} \) in terms of sine and cosine.

\[ \sqrt{3} - i = Re^{i\theta} = R(\cos \theta + i \sin \theta) = R \cos \theta + iR \sin \theta \]

Match the coefficients to obtain a system of equations for \( R \) and \( \theta \).

\[ R \cos \theta = \sqrt{3} \quad (1) \]
\[ R \sin \theta = -1 \quad (2) \]

To determine \( R \), square both sides of each equation

\[ R^2 \cos^2 \theta = 3 \]
\[ R^2 \sin^2 \theta = 1 \]

and then add the respective sides.

\[ R^2 \cos^2 \theta + R^2 \sin^2 \theta = 3 + 1 \]
\[ R^2 = 4 \]
\[ R = 2 \]

Divide both sides of equation (2) by the respective sides of equation (1).

\[ \tan \theta = -\frac{1}{\sqrt{3}} \]
\[ \theta = -\frac{\pi}{6} + 2n\pi, \quad n = 0, \pm1, \pm2, \ldots \]

Note that adding any multiple of 2\( \pi \) does not change the point’s position on the \( xy \)-plane. Therefore,

\[ \sqrt{3} - i = 2e^{i(-\frac{\pi}{6} + 2n\pi)}. \]