Problem 6

In each of Problems 1 through 6, express the given complex number in the form
\[ R(\cos \theta + i \sin \theta) = Re^{i\theta}. \]

Solution

Use Euler’s formula to write \( e^{i\theta} \) in terms of sine and cosine.

\[ -1 - i = Re^{i\theta} \]
\[ = R(\cos \theta + i \sin \theta) \]
\[ = R \cos \theta + iR \sin \theta \]

Match the coefficients to obtain a system of equations for \( R \) and \( \theta \).

\[ R \cos \theta = -1 \quad (1) \]
\[ R \sin \theta = -1 \quad (2) \]

To determine \( R \), square both sides of each equation

\[ R^2 \cos^2 \theta = 1 \]
\[ R^2 \sin^2 \theta = 1 \]

and then add the respective sides.

\[ R^2 \cos^2 \theta + R^2 \sin^2 \theta = 1 + 1 \]
\[ R^2 = 2 \]
\[ R = \sqrt{2} \]

Divide both sides of equation (2) by the respective sides of equation (1).

\[ \tan \theta = 1 \]
\[ \theta = \frac{5\pi}{4} + 2n\pi, \quad n = 0, \pm 1, \pm 2, \ldots \]

Note that adding any multiple of \( 2\pi \) does not change the point’s position on the \( xy \)-plane. Therefore,

\[ -1 - i = \sqrt{2}e^{i(5\pi/4+2n\pi)} \].