Problem 27

In each of Problems 21 through 27, rewrite the given expression as a sum whose generic term involves x^n.

$$x\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} + \sum_{n=0}^{\infty} a_n x^n$$

Solution

Bring x into the summand.

$$\sum_{n=2}^{\infty} n(n-1)a_n x^{n-1} + \sum_{n=0}^{\infty} a_n x^n$$

Because of the $n-1$ factor, the first sum can be started at $n=1$.

$$\sum_{n=1}^{\infty} n(n-1)a_n x^{n-1} + \sum_{n=0}^{\infty} a_n x^n$$

Substitute $k = n-1$ in the first sum and substitute $k = n$ in the second sum.

$$\sum_{k+1=1}^{\infty} (k+1)ka_{k+1}x^k + \sum_{k=0}^{\infty} a_k x^k$$

Solve for k.

$$\sum_{k=0}^{\infty} k(k+1)a_{k+1}x^k + \sum_{k=0}^{\infty} a_k x^k$$

Combine the two sums.

$$\sum_{k=0}^{\infty}[k(k+1)a_{k+1}x^k + a_k x^k]$$

Factor x^k.

$$\sum_{k=0}^{\infty}[k(k+1)a_{k+1} + a_k]x^k$$

As k is only a dummy index, it can be replaced with n.

$$\sum_{n=0}^{\infty}[n(n+1)a_{n+1} + a_n]x^n$$