Problem 1

In each of Problems 1 through 8, determine the radius of convergence of the given power series.

\[ \sum_{n=0}^{\infty} (x - 3)^n \]

Solution

Apply the ratio test to determine the condition in which the provided series converges.

\[
\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \left| \frac{(x - 3)^{n+1}}{(x - 3)^n} \right| \\
= \lim_{n \to \infty} |x - 3| \\
= |x - 3|
\]

According to this test, the series is

\[
\begin{cases}
\text{convergent} & \text{if } |x - 3| < 1 \\
\text{unknown} & \text{if } |x - 3| = 1 \\
\text{divergent} & \text{if } |x - 3| > 1
\end{cases}
\]

From the condition of convergence, which can also be written as \(-1 < x - 3 < 1\), or \(2 < x < 4\), we see that the center of convergence is at \(x = 3\) and the radius of convergence is 1.