Problem 2

In each of Problems 1 through 8, determine the radius of convergence of the given power series.

\[ \sum_{n=0}^{\infty} \frac{n^2}{2^n} x^n \]

Solution

Apply the ratio test to determine the condition in which the provided series converges.

\[
\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \left| \frac{\frac{n+1}{2^{n+1}} x^{n+1}}{\frac{n}{2^n} x^n} \right| = \lim_{n \to \infty} \left| \frac{1 + \frac{1}{n}}{2} |x| \right| = \frac{1}{2} \lim_{n \to \infty} \left( 1 + \frac{1}{n} \right) |x| = \frac{1}{2} |x|
\]

According to this test, the series is

\[
\begin{cases}
\text{convergent} & \text{if } \frac{1}{2} |x| < 1 \\
\text{unknown} & \text{if } \frac{1}{2} |x| = 1 \\
\text{divergent} & \text{if } \frac{1}{2} |x| > 1
\end{cases}
\]

From the condition of convergence, which can also be written as \( |x| < 2 \), or \(-2 < x < 2\), we see that the center of convergence is at \( x = 0 \) and the radius of convergence is \( 2 \).