Problem 24

In each of Problems 21 through 27, rewrite the given expression as a sum whose generic term involves \(x^n\).

\[
(1 - x^2) \sum_{n=2}^{\infty} n(n - 1)a_n x^{n-2}
\]

Solution

Expand this term.

\[
\sum_{n=2}^{\infty} n(n - 1)a_n x^{n-2} - x^2 \sum_{n=2}^{\infty} n(n - 1)a_n x^{n-2}
\]

Bring \(x^2\) into the summand.

\[
\sum_{n=2}^{\infty} n(n - 1)a_n x^{n-2} - \sum_{n=2}^{\infty} n(n - 1)a_n x^n
\]

Substitute \(k = n - 2\) in the first sum and substitute \(k = n\) in the second sum.

\[
\sum_{k+2=2}^{\infty} (k + 2)(k + 1)a_{k+2}x^k - \sum_{k=2}^{\infty} k(k - 1)a_kx^k
\]

Solve for \(k\).

\[
\sum_{k=0}^{\infty} (k + 2)(k + 1)a_{k+2}x^k - \sum_{k=0}^{\infty} k(k - 1)a_kx^k
\]

Because of the factors, \(k\) and \(k - 1\), the second sum can be started at \(k = 0\).

\[
\sum_{k=0}^{\infty} (k + 2)(k + 1)a_{k+2}x^k - \sum_{k=0}^{\infty} k(k - 1)a_kx^k
\]

Combine the two sums.

\[
\sum_{k=0}^{\infty} [(k + 2)(k + 1)a_{k+2}x^k - k(k - 1)a_kx^k]
\]

Factor \(x^k\).

\[
\sum_{k=0}^{\infty} [(k + 2)(k + 1)a_{k+2} - k(k - 1)a_k]x^k
\]

As \(k\) is only a dummy index, it can be replaced with \(n\).

\[
\sum_{n=0}^{\infty} [(n + 2)(n + 1)a_{n+2} - n(n - 1)a_n]x^n
\]

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