Problem 3

In each of Problems 1 through 8, determine the radius of convergence of the given power series.

\[ \sum_{n=0}^{\infty} \frac{x^{2n}}{n!} \]

Solution

Apply the ratio test to determine the condition in which the provided series converges.

\[
\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \left| \frac{x^{2(n+1)}}{(n+1)!} \right| \frac{n!}{x^{2n}} \frac{1}{n!} x^{2n} \\
= \lim_{n \to \infty} \left| \frac{n+1}{n} \frac{x^2}{x^{2n}} \right| \\
= \lim_{n \to \infty} \left| \frac{1}{n+1} x^2 \right| \\
= 0x^2 \\
= 0
\]

According to this test, the series is

\[
\begin{cases} 
\text{convergent} & \text{if } 0x^2 < 1 \\
\text{unknown} & \text{if } 0x^2 = 1 \\
\text{divergent} & \text{if } 0x^2 > 1 
\end{cases}
\]

From the condition of convergence, which can also be written as \(|x| < \sqrt{1/0} = \infty\), or \(-\infty < x < \infty\), we see that the center of convergence is at \(x = 0\) and the radius of convergence is \(\infty\).