# Problem 2

In each of Problems 1 through 4, determine $\phi''(x_0)$, $\phi'''(x_0)$, and $\phi^{(4)}(x_0)$ for the given point $x_0$ if $y = \phi(x)$ is a solution of the given initial value problem.

$$y'' + (\sin x)y' + (\cos x)y = 0; \qquad y(0) = 0, \quad y'(0) = 1$$

---

### Solution

Solve the ODE for $y''$.

$$y'' = -(\sin x)y' - (\cos x)y \tag{1}$$

Plug in $x = 0$.

$$y''(0) = -(\sin 0)y'(0) - (\cos 0)y(0) = 0$$

Differentiate both sides of equation (1) with respect to $x$.

$$y''' = -(\cos x)y' - (\sin x)y'' + (\sin x)y - (\cos x)y' \tag{2}$$

Plug in $x = 0$.

$$y'''(0) = -(\cos 0)y'(0) - (\sin 0)y''(0) + (\sin 0)y(0) - (\cos 0)y'(0) = -2y'(0) = -2$$

Differentiate both sides of equation (2) with respect to $x$.

$$y^{(4)} = (\sin x)y' - (\cos x)y'' - (\cos x)y'' - (\sin x)y''' + (\cos x)y + (\sin x)y' + (\sin x)y' - (\cos x)y''$$

Plug in $x = 0$.

$$\begin{aligned} y^{(4)}(0) = (\sin 0)y'(0) - (\cos 0)y''(0) - (\cos 0)y''(0) - (\sin 0)y'''(0) + (\cos 0)y(0) \\ + (\sin 0)y'(0) + (\sin 0)y'(0) - (\cos 0)y''(0) = -3y''(0) + y(0) = 0 \end{aligned}$$