Problem 9

Determine a lower bound for the radius of convergence of series solutions about the given x_0 for each of the differential equations in Problems 1 through 14 of Section 5.2.

Solution

Problem 1

$$y'' - y = 0, \qquad x_0 = 0$$

The coefficient of y'' is 1. It has no zeros, so the lower bound for the radius of convergence is ∞.

Problem 2

$$y'' - xy' - y = 0, \qquad x_0 = 0$$

The coefficient of y'' is 1. It has no zeros, so the lower bound for the radius of convergence is ∞.

Problem 3

$$y'' - xy' - y = 0, \qquad x_0 = 1$$

The coefficient of y'' is 1. It has no zeros, so the lower bound for the radius of convergence is ∞.

Problem 4

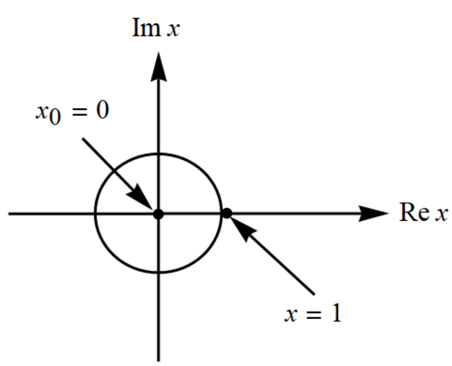
$$y'' + k^2x^2y = 0, \qquad x_0 = 0, \quad k \text{ a constant}$$

The coefficient of y'' is 1. It has no zeros, so the lower bound for the radius of convergence is ∞.

Problem 5

$$(1 - x)y'' + y = 0, \qquad x_0 = 0$$

Its zero is located at $x = 1$. Plot it in the complex plane and expand a circle centered at x_0 as much as possible until it intersects $x = 1$.

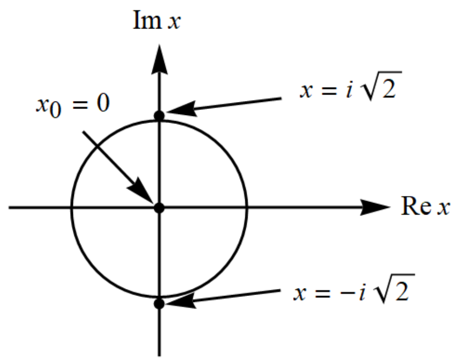


If $x_0 = 0$, the lower bound for the radius of convergence is 1.

Problem 6

$$(2 + x^2)y'' - xy' + 4y = 0, \qquad x_0 = 0$$

The coefficient of y'' is $2 + x^2$. Its zeros are located at $x = -i\sqrt{2}$ and $x = i\sqrt{2}$. Plot these in the complex plane and expand a circle centered at x_0 as much as possible until it intersects one of them.



If $x_0 = 0$, the lower bound for the radius of convergence is $\sqrt{2}$.

Problem 7

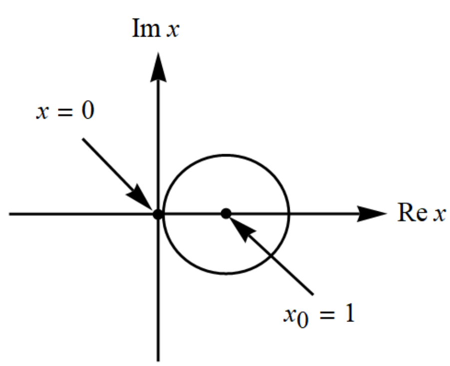
$$y'' + xy' + 2y = 0, \qquad x_0 = 0$$

The coefficient of y'' is 1. It has no zeros, so the lower bound for the radius of convergence is ∞.

Problem 8

$$xy'' + y' + xy = 0, \qquad x_0 = 1$$

The coefficient of y'' is x. Its zero is located at $x = 0$. Plot it in the complex plane and expand a circle centered at x_0 as much as possible until it intersects $x = 0$.

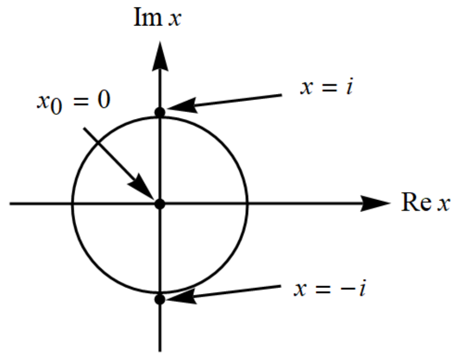


If $x_0 = 1$, the lower bound for the radius of convergence is 1.

Problem 9

$$(1 + x^2)y'' - 4xy' + 6y = 0, \qquad x_0 = 0$$

The coefficient of y'' is $1 + x^2$. Its zeros are located at $x = -i$ and $x = i$. Plot these in the complex plane and expand a circle centered at x_0 as much as possible until it intersects one of them.

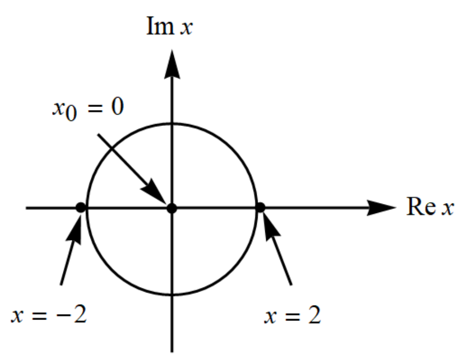


If $x_0 = 0$, the lower bound for the radius of convergence is 1.

Problem 10

$$(4 - x^2)y'' + 2y = 0, \qquad x_0 = 0$$

The coefficient of y'' is $4 - x^2$. Its zeros are located at $x = -2$ and $x = 2$. Plot these in the complex plane and expand a circle centered at x_0 as much as possible until it intersects one of them.

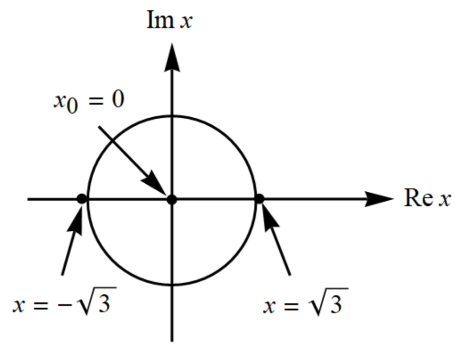


If $x_0 = 0$, the lower bound for the radius of convergence is 2.

Problem 11

$$(3 - x^2)y'' - 3xy' - y = 0, \qquad x_0 = 0$$

The coefficient of y'' is $3 - x^2$. Its zeros are located at $x = -\sqrt{3}$ and $x = \sqrt{3}$. Plot these in the complex plane and expand a circle centered at x_0 as much as possible until it intersects one of them.

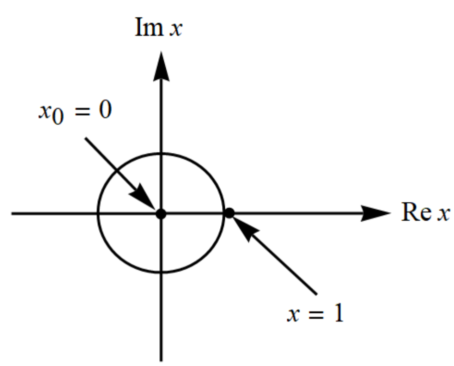


If $x_0 = 0$, the lower bound for the radius of convergence is $\sqrt{3}$.

Problem 12

$$(1 - x)y'' + xy' - y = 0, \qquad x_0 = 0$$

The coefficient of y'' is $1 - x$. Its zero is located at $x = 1$. Plot it in the complex plane and expand a circle centered at x_0 as much as possible until it intersects $x = 0$.



If $x_0 = 0$, the lower bound for the radius of convergence is 1.

Problem 13

$$2y'' + xy' + 3y = 0, \qquad x_0 = 0$$

The coefficient of y'' is 2. It has no zeros, so the lower bound for the radius of convergence is ∞.

Problem 14

$$2y'' + (x + 1)y' + 3y = 0, \qquad x_0 = 2$$

The coefficient of y'' is 2. It has no zeros, so the lower bound for the radius of convergence is ∞.