Problem 1

In each of Problems 1 through 4, determine \( \phi''(x_0) \), \( \phi'''(x_0) \), and \( \phi^{(4)}(x_0) \) for the given point \( x_0 \) if \( y = \phi(x) \) is a solution of the given initial value problem.

\[
y'' + xy' + y = 0; \quad y(0) = 1, \quad y'(0) = 0
\]

Solution

Solve the ODE for \( y'' \).

\[
y'' = -xy' - y \tag{1}
\]

Plug in \( x = 0 \).

\[
y''(0) = -(0)y'(0) - y(0) = -1
\]

Differentiate both sides of equation (1) with respect to \( x \).

\[
y''' = -y' - xy'' - y' \tag{2}
\]

Plug in \( x = 0 \).

\[
y'''(0) = -y'(0) - (0)y''(0) - y'(0) = 0
\]

Differentiate both sides of equation (2) with respect to \( x \).

\[
y^{(4)} = -y'' - y'' - xy''' - y''
\]

Plug in \( x = 0 \).

\[
y^{(4)}(0) = -y''(0) - y''(0) - (0)y'''(0) - y''(0) = 1 + 1 + 1 = 3
\]