Problem 18

First Order Equations. The series methods discussed in this section are directly applicable to the first order linear differential equation $P(x)y' + Q(x)y = 0$ at a point $x_0$, if the function $p = Q/P$ has a Taylor series expansion about that point. Such a point is called an ordinary point, and further, the radius of convergence of the series $y = \sum_{n=0}^{\infty} a_n (x - x_0)^n$ is at least as large as the radius of convergence of the series for $Q/P$. In each of Problems 16 through 21, solve the given differential equation by a series in powers of $x$ and verify that $a_0$ is arbitrary in each case. Problems 20 and 21 involve nonhomogeneous differential equations to which series methods can be easily extended. Where possible, compare the series solution with the solution obtained by using the methods of Chapter 2.

$$y' = e^{x^2} y, \quad \text{three terms only}$$