Problem 6

In each of Problems 5 through 8, determine a lower bound for the radius of convergence of series solutions about each given point $x_0$ for the given differential equation.

$$(x^2 - 2x - 3)y'' + xy' + 4y = 0; \quad x_0 = 4, \quad x_0 = -4, \quad x_0 = 0$$

Solution

The coefficient of $y''$ is $x^2 - 2x - 3$. Its zeros are located at $x = -1$ and $x = 3$. Plot these in the complex plane and expand a circle centered at $x_0$ as much as possible before it intersects one of them.

If $x_0 = 4$, the lower bound for the radius of convergence is 1. If $x_0 = -4$, the lower bound for the radius of convergence is 3. If $x_0 = 0$, the lower bound for the radius of convergence is 1.