Problem 1

In each of Problems 1 through 12, determine the general solution of the given differential equation that is valid in any interval not including the singular point.

$$x^2y'' + 4xy' + 2y = 0$$

Solution

This is a homogeneous equidimensional (Euler) ODE, so the solution is of the form $y = x^r$.

$$y = x^r \quad \rightarrow \quad y' = rx^{r-1} \quad \rightarrow \quad y'' = r(r-1)x^{r-2}$$

Substitute these expressions into the ODE.

$$x^2r(r-1)x^{r-2} + 4xrx^{r-1} + 2x^r = 0$$

$$r(r-1)x^r + 4rx^r + 2x^r = 0$$

Divide both sides by x^r.

$$r(r-1) + 4r + 2 = 0$$

Solve for r.

$$r^2 + 3r + 2 = 0$$

$$(r+2)(r+1) = 0$$

$$r = \{-2, -1\}$$

Two solutions to the ODE are $y = x^{-2}$ and $y = x^{-1}$. According to the principle of superposition, the general solution is a linear combination of these two. Therefore,

$$y(x) = C_1x^{-2} + C_2x^{-1}.$$