

## Problem 17

In each of Problems 17 through 34, find all singular points of the given equation and determine whether each one is regular or irregular.

$$xy'' + (1 - x)y' + xy = 0$$

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### Solution

The coefficient of  $y''$  has a zero at  $x = 0$ , which means  $x = 0$  is a singular point. To determine whether it is regular or irregular, divide both sides of the ODE by  $x$

$$y'' + \left(\frac{1}{x} - 1\right)y' + (1)y = 0$$

and compute the following limits.

$$\lim_{x \rightarrow 0} x \left(\frac{1}{x} - 1\right) = \lim_{x \rightarrow 0} (1 - x) = 1$$
$$\lim_{x \rightarrow 0} x^2(1) = 0$$

Since both limits are finite,  $x = 0$  is a regular singular point.