

Problem 22

In each of Problems 17 through 34, find all singular points of the given equation and determine whether each one is regular or irregular.

$$x^2y'' + xy' + (x^2 - \nu^2)y = 0, \quad \text{Bessel equation}$$

Solution

The coefficient of y'' has a zero at $x = 0$, which means $x = 0$ is a singular point. To determine whether it is regular or irregular, divide both sides of the ODE by x^2

$$y'' + \left(\frac{1}{x}\right)y' + \left(1 - \frac{\nu^2}{x^2}\right)y = 0$$

and compute the following limits.

$$\lim_{x \rightarrow 0} x \left(\frac{1}{x}\right) = \lim_{x \rightarrow 0} 1 = 1$$
$$\lim_{x \rightarrow 0} x^2 \left(1 - \frac{\nu^2}{x^2}\right) = \lim_{x \rightarrow 0} (x^2 - \nu^2) = -\nu^2$$

Because both limits as $x \rightarrow 0$ are finite, $x = 0$ is a regular singular point.