

Problem 21

In each of Problems 21 through 24, find the Laplace transform of the given function.

$$f(t) = \begin{cases} 1, & 0 \leq t < \pi \\ 0, & \pi \leq t < \infty \end{cases}$$

Solution

The Laplace transform of a function $f(t)$ is defined as

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt.$$

Split up the integral over the intervals that $f(t)$ is defined on.

$$\begin{aligned} \mathcal{L}\{f(t)\} &= \int_0^{\pi} e^{-st}(1) dt + \int_{\pi}^{\infty} e^{-st}(0) dt \\ &= \int_0^{\pi} e^{-st} dt \\ &= -\frac{1}{s} e^{-st} \Big|_0^{\pi} \\ &= -\frac{1}{s} (e^{-\pi s} - e^0) \\ &= \frac{1}{s} (1 - e^{-\pi s}) \end{aligned}$$

Below is a side-by-side comparison of the function $f(t)$ and its Laplace transform $F(s)$.

