Problem 28

In each of Problems 25 through 28, determine whether the given integral converges or diverges.

$$\int_0^\infty e^{-t}\cos t\,dt$$

Solution

$$\begin{aligned}
\int_0^\infty e^{-t}\cos t\,dt &= \int_0^\infty \frac{d}{dt}(-e^{-t})\cos t\,dt \\
&= (-e^{-t})\cos t\Big|_0^\infty - \int_0^\infty (-e^{-t})(-\sin t)\,dt \\
&= 1 - \int_0^\infty e^{-t}\sin t\,dt \\
&= 1 - \int_0^\infty \frac{d}{dt}(-e^{-t})\sin t\,dt \\
&= 1 - \left[(-e^{-t})\sin t\Big|_0^\infty - \int_0^\infty (-e^{-t})\cos t\,dt\right] \\
&= 1 - \left[\int_0^\infty e^{-t}\cos t\,dt\right]
\end{aligned}$$

Bring this second term to the left side.

$$2\int_0^\infty e^{-t}\cos t\,dt = 1$$

Dividing both sides by 2, we see that the integral converges.

$$\int_0^\infty e^{-t}\cos t\,dt = \frac{1}{2}$$