Problem 26

In each of Problems 25 through 28, determine whether the given integral converges or diverges.

\[ \int_0^\infty te^{-t} \, dt \]

Solution

\[
\int_0^\infty te^{-t} \, dt = \int_0^\infty t \frac{d}{dt} (-e^{-t}) \, dt \\
= t(-e^{-t}) \bigg|_0^\infty - \int_0^\infty (1)(-e^{-t}) \, dt \\
= \int_0^\infty e^{-t} \, dt \\
= -e^{-t} \bigg|_0^\infty \\
= e^0 - e^\infty \\
= 1
\]

The integral converges.