Problem 31

Consider the Laplace transform of $t^p$, where $p > -1$.

(a) Referring to Problem 30, show that

$$\mathcal{L}\{t^p\} = \int_0^\infty e^{-st}t^p \, dt = \frac{1}{s^{p+1}} \int_0^\infty e^{-x}x^p \, dx$$

$$= \Gamma(p + 1)/s^{p+1}, \quad s > 0.$$ 

(b) Let $p$ be a positive integer $n$ in part (a); show that

$$\mathcal{L}\{t^n\} = n!/s^{n+1}, \quad s > 0.$$ 

(c) Show that

$$\mathcal{L}\{t^{-1/2}\} = \frac{2}{\sqrt{s}} \int_0^\infty e^{-x^2} \, dx, \quad s > 0.$$ 

It is possible to show that

$$\int_0^\infty e^{-x^2} \, dx = \frac{\sqrt{\pi}}{2};$$

hence

$$\mathcal{L}\{t^{-1/2}\} = \sqrt{\pi}/s, \quad s > 0.$$ 

(d) Show that

$$\mathcal{L}\{t^{1/2}\} = \sqrt{\pi}/(2s^{3/2}), \quad s > 0.$$