Problem 7

In each of Problems 1 through 10, find the inverse Laplace transform of the given function.

$$F(s) = \frac{2s+1}{s^2 - 2s + 2}$$

Solution

Complete the square in the denominator.

$$\begin{aligned} F(s) &= \frac{2s+1}{s^2 - 2s + 1 + 2 - 1} \\ &= \frac{2s+1}{(s-1)^2 + 1} \end{aligned}$$

Make it so that the numerator also has $s - 1$ to get $F(s)$ in terms of known transforms.

$$\begin{aligned} F(s) &= \frac{2s - 2 + 3}{(s-1)^2 + 1} \\ &= \frac{2s-2}{(s-1)^2+1} + \frac{3}{(s-1)^2+1} \\ &= 2\frac{s-1}{(s-1)^2+1} + 3\frac{1}{(s-1)^2+1} \end{aligned}$$

Take the inverse Laplace transform to get $f(t)$.

$$\begin{aligned} \mathcal{L}^{-1}\{F(s)\} &= \mathcal{L}^{-1}\left\{2\frac{s-1}{(s-1)^2+1} + 3\frac{1}{(s-1)^2+1}\right\} \\ f(t) &= 2\mathcal{L}^{-1}\left\{\frac{s-1}{(s-1)^2+1}\right\} + 3\mathcal{L}^{-1}\left\{\frac{1}{(s-1)^2+1}\right\} \\ &= 2e^t \cos t + 3e^t \sin t \end{aligned}$$