Problem 27

In each of Problems 26 through 29, use the results of Problem 25 to find the inverse Laplace transform of the given function.

$$F(s) = \frac{2s+1}{4s^2+4s+5}$$

Solution

Make it so that $2s+1$ is in the denominator as well.

$$\begin{aligned} F(s) &= \frac{2s+1}{4s^2+4s+1+5-1} \\ &= \frac{2s+1}{(2s+1)^2+4} \end{aligned}$$

Apply the two transforms,

$$\mathcal{L}\{\cos at\} = \frac{s}{s^2+a^2} \quad \text{and} \quad \mathcal{L}\left\{\frac{1}{a}e^{-bt/a}f\left(\frac{t}{a}\right)\right\} = F(as+b),$$

together to solve this problem.

$$\begin{aligned} f(t) &= \mathcal{L}^{-1}\{F(s)\} \\ &= \mathcal{L}^{-1}\left\{\frac{2s+1}{(2s+1)^2+4}\right\} \\ &= \frac{1}{2}e^{-t/2}\cos\left(\frac{2t}{2}\right) \\ &= \frac{1}{2}e^{-t/2}\cos t \end{aligned}$$